Problem 1.3

Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3$ A, $q(0) = 1$ C

(b) $i(t) = (2t + 5)$ mA, $q(0) = 0$

(c) $i(t) = 20\cos(10t + \pi/6)$ μA, $q(0) = 2$ μC

(d) $i(t) = 10e^{-30t}\sin 40t$ A, $q(0) = 0$

Solution

Begin with the basic equation relating current and charge and then integrate both sides with respect to time.

$$i(t) = \frac{dq}{dt}$$

$$\int i(t)\,dt = \int \frac{dq}{dt}\,dt$$

$$\int i(t)\,dt = q(t)$$

Part (a)

Find the indefinite integral of the given function: $i(t) = 3$ A subject to $q(0) = 1$ C.

$$q(t) = \int i(t)\,dt = \int (3)\,dt \text{ A} = (3t + C) \text{ C}$$

Apply the initial condition to determine C.

$$q(0) = C = 1$$

Therefore,

$$q(t) = (3t + 1) \text{ C}.$$

Part (b)

Find the indefinite integral of the given function: $i(t) = (2t + 5)$ mA subject to $q(0) = 0$.

$$q(t) = \int i(t)\,dt = \int (2t + 5)\,dt \text{ mA} = (t^2 + 5t + C) \text{ mC}$$

Apply the initial condition to determine C.

$$q(0) = C = 0$$

Therefore,

$$q(t) = (t^2 + 5t) \text{ mC}.$$

Part (c)

Find the indefinite integral of the given function: $i(t) = 20\cos(10t + \pi/6)$ μA subject to $q(0) = 2$ μC.

$$q(t) = \int i(t)\,dt = \int 20\cos(10t + \pi/6)\,dt\ \mu\text{A} = [2\sin(10t + \pi/6) + C]\ \mu\text{C}$$

Apply the initial condition to determine C.

$$q(0) = 2\sin(\pi/6) + C = 2$$

$$2\left(\frac{1}{2}\right) + C = 2$$

$$1 + C = 2$$

$$C = 1$$

Therefore,

$$q(t) = [2\sin(10t + \pi/6) + 1]\ \mu\text{C}.$$

Part (d)

Find the indefinite integral of the given function: $i(t) = 10e^{-30t}\sin 40t$ A subject to $q(0) = 0$.

$$\begin{aligned}
q(t) = \int i(t)\,dt &= \int 10e^{-30t}\sin 40t\,dt\ \text{A} \\
&= 10\int e^{-30t}\sin 40t\,dt\ \text{A} \\
&= 10\int e^{-30t}\left(-\frac{1}{40}\frac{d}{dt}\cos 40t\right)dt\ \text{A} \\
&= -\frac{1}{4}\int e^{-30t}\frac{d}{dt}(\cos 40t)\,dt\ \text{A} \\
&= -\frac{1}{4}\left[e^{-30t}\cos 40t - \int \frac{d}{dt}(e^{-30t})\cos 40t\,dt\right]\text{A} \\
&= -\frac{1}{4}\left[e^{-30t}\cos 40t - \int (-30e^{-30t})\cos 40t\,dt\right]\text{A} \\
&= -\frac{1}{4}\left(e^{-30t}\cos 40t + 30\int e^{-30t}\cos 40t\,dt\right)\text{A} \\
&= -\frac{1}{4}\left[e^{-30t}\cos 40t + 30\int e^{-30t}\left(\frac{1}{40}\frac{d}{dt}\sin 40t\right)dt\right]\text{A} \\
&= -\frac{1}{4}\left[e^{-30t}\cos 40t + \frac{3}{4}\int e^{-30t}\frac{d}{dt}(\sin 40t)\,dt\right]\text{A} \\
&= -\frac{1}{4}\left[e^{-30t}\cos 40t + \frac{3}{4}\left(e^{-30t}\sin 40t - \int \frac{d}{dt}(e^{-30t})\sin 40t\,dt\right)\right]\text{A}
\end{aligned}$$

Consequently,

$$10\int e^{-30t}\sin 40t\,dt = -\frac{1}{4}\left[e^{-30t}\cos 40t + \frac{3}{4}\left(e^{-30t}\sin 40t - \int(-30e^{-30t})\sin 40t\,dt\right)\right]$$

$$10\int e^{-30t}\sin 40t\,dt = -\frac{1}{4}\left(e^{-30t}\cos 40t + \frac{3}{4}e^{-30t}\sin 40t + \frac{45}{2}\int e^{-30t}\sin 40t\,dt\right)$$

$$10\int e^{-30t}\sin 40t\,dt = -\frac{1}{4}e^{-30t}\cos 40t - \frac{3}{16}e^{-30t}\sin 40t - \frac{45}{8}\int e^{-30t}\sin 40t\,dt$$

$$\left(10 + \frac{45}{8}\right)\int e^{-30t}\sin 40t\,dt = -\frac{1}{4}e^{-30t}\cos 40t - \frac{3}{16}e^{-30t}\sin 40t$$

$$\frac{125}{8}\int e^{-30t}\sin 40t\,dt = -\frac{1}{4}e^{-30t}\cos 40t - \frac{3}{16}e^{-30t}\sin 40t$$

$$\int e^{-30t}\sin 40t\,dt = -\frac{2}{125}e^{-30t}\cos 40t - \frac{3}{250}e^{-30t}\sin 40t,$$

which means

$$q(t) = 10\int e^{-30t}\sin 40t\,dt\text{ A} = \left(-\frac{4}{25}e^{-30t}\cos 40t - \frac{3}{25}e^{-30t}\sin 40t + C\right)\text{A}.$$

Apply the initial condition to determine C.

$$q(0) = -\frac{4}{25} + C = 0 \quad\rightarrow\quad C = \frac{4}{25}$$

Therefore,

$$q(t) = \left(-\frac{4}{25}e^{-30t}\cos 40t - \frac{3}{25}e^{-30t}\sin 40t + \frac{4}{25}\right)\text{A}.$$